AIChE

Multiechelon Supply Chain Planning with Sequence-Dependent Changeovers and Price Elasticity of Demand under Uncertainty

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DOI 10.1002/aic.13732
Published online February 1, 2012 in Wiley Online Library (wileyonlinelibrary.com).

An optimization framework is proposed for a multiechelon multiproduct process supply chain planning under demand uncertainty considering inventory deviation and price fluctuation. In this problem, the sequence-dependent changeovers occur at the production plants, and price elasticity of demand is considered at the markets. Based on the classic formulation of travelling salesman problem (TSP), a mixed-integer liner programming (MILP) is developed, whose objective function considers the profit, inventory deviations from the desired trajectories and price changes simultaneously. Model predictive control (MPC) approach is adopted to tackle the uncertain issues, as well as the inventory and price maintenance. The applicability of the proposed model and approach was illustrated by solving a supply chain example. Some issues, including length of the control horizon, price elasticity of demand, weights, inventory trajectories, and changeovers, are discussed based on the computational results. © 2012 American Institute of Chemical Engineers *AIChE J,* 58: 3390–3403, 2012
Keywords: supply chain management, price elasticity of demand, inventory control, pricing, model predictive control, sequence-dependent changeovers

Introduction

Supply chain management is an important issue in the process systems engineering.[1–3] A number of optimization-based models have been presented to model the multisite production and distribution planning of process supply chains in both deterministic[4–8] and uncertain environments.[9–14]

During production, when switching from one product, product family, or production mode to another one, changeovers usually occur for the cleanup, setup and preparation work. If the production changeovers are dependent on the sequence of the products, product families or production modes involved, the production sequence may affect the capacity usage of the plants, as well as the efficiency of the whole supply chain. Thus, the sequence-dependent changeovers is an important issue and have been largely investigated in the literate on production planning and scheduling.[15–23] A review by Zhu and Wilhelm[24] gave a detailed overview of scheduling problems involving sequence-dependent changeovers. So far, A few literature models have considered the production and distribution planning at multisite supply chains with sequence-dependent changeovers. Kallrath and coworkers[4,25] modeled the sequence-dependent changeovers in multisite production networks using a discrete-time mixed-integer linear programming (MILP) formulation. Bok et al.[26] developed an MILP model for multisite continuous flexible process supply chain networks with sequence-dependent changeovers. Perea-López et al.[27] used a special case of the state-task-network (STN) structure to model the production planning with sequence-dependent changeovers. They used model predictive control (MPC) approach to implement the proposed MILP model, in which changeover times were neglected. Nishi et al.[28] proposed an autonomous decentralized optimization system for multistage production processes, including material requirement planning subsystem, scheduling subsystem and distribution planning subsystem. The proposed MILP model considered an objective of minimization of the total cost, including the sequence-dependent changeover cost at the production stage. Recently, Terrazas-Moreno et al.[29] developed both temporal and spatial Lagrangean decomposition approaches for a multisite, multiperiod, and multiproduct planning problem in with sequence-dependent changeovers, which was formulated as an MILP problem.

The purpose of our work is to propose an optimization framework for the production and distribution planning of a multiechelon multiproduct supply chain under demand

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uncertainty, to simultaneously consider sequence-dependent changeovers, price elasticity of demand and inventory deviations from the desired trajectories. Extending our previous work[18–20] on the production planning and scheduling, an MILP optimization model is developed using classic travelling salesman problem (TSP) formulations to model the optimal production sequences without subtours considering the sequence-dependent changeovers.

The pricing strategy is another important issue to the supply chain, especially when the price elasticity of demand is high, i.e., the price has a significant effect on the product demands. Thus, how to make the correct pricing decisions is crucial in the supply chain management. Although some literature work has been done to investigate the supply chains with the price elasticity of demand,[30–37] none of the aforementioned work has considered price fluctuations, which is one of the main reasons for the bullwhip effect in the supply chains.[38–40]

In this work, we aim to adopt an MPC approach to implement the proposed model, similar to the work of Perea-López et al.[27] which applied MPC approach to implement the proposed optimization model for the optimal decisions to maximize profits. The MPC approach is used to implement the novel MILP formulation of the supply chain planning problem considering not only the profit, but also the maintenance of the inventory levels and price levels in the objective function. To the best of our knowledge, it is the first work to use MILP- and MPC-based approach for supply chain planning problem with sequence-dependent changeovers considering profit, inventory deviation and price fluctuation simultaneously.

The rest of this article is organized as follows. The problem is described in the next section. Then the problem is formulated as an MILP model. This is followed by the description of MPC approach. Then a numerical example is considered, and the results and discussion are presented. Finally, the concluding remarks are given.

Problem Description

We consider a supply chain network with three echelons, including plants, DCs and markets (see example in Figure 1). The whole planning horizon of the problem is divided into multiple time periods. In the plants, multiple products are produced with the occurrences of sequence-dependent changeovers. The processed final products are shipped to several DCs. Then the final products are transported from DCs to the markets for sales. It is assumed that all the deliveries are done at the end of each time period. When the sales volume of a product is less than its actual demand, the unmet demand is lost. The costs of production, transportation, changeovers and unmet demands occur during the aforementioned processes.

Each final product is stored at all suitable echelon nodes including plants, DCs and markets. There are inventory trajectories for products at all echelon nodes in different time periods. The aim of the inventory management is to control the inventory to be as close to the inventory trajectory as possible, i.e., to keep the inventory deviation from the inventory trajectory as small as possible. In this case, the inventory cost incurred will not be included in the total cost. Otherwise, the profit maximization, which results in inventory cost minimization, will conflict with the inventory control.

The demands of each product in each market are affected by the product's prices in the market by the price elasticity of demand. For each product, there is an initial demand in a time period corresponding to the product's initial price at

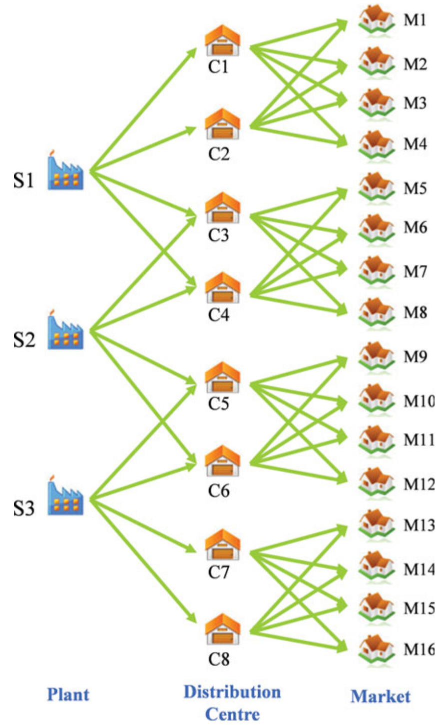

Figure 1. The structure of the supply chain in the example.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

each market. In each time period, there are several price levels to be selected for each product at one market. If the selected price is higher than the initial price, the actual final demand will become lower than the initial demand; while if a lower price level is selected, the actual final demand will be higher than the initial demand. In this problem, the initial demands are uncertain. Before the initial demands are realized, the forecasts of initial demands can be predicted. The initial demands are realized at the beginning of each time period. When the pricing decisions are made, actual final demands can be known accordingly. In order to maintain a stable price level to avoid large price fluctuations at the markets, the price changes between two consecutive time periods are considered as well.

In the supply chain production, distribution and inventory planning problem, the following are given:

- Echelon nodes (plants, DCs and markets) and their suitabilities and connections between them;
- unit production costs and changeover costs and times;
- unit transportation costs;
- unit inventory costs and inventory trajectories;
- unit unmet demand costs;
- initial demand forecast;
- available product price levels and price elasticity coefficients;
- minimum and maximum allowed inventories;

to determine

- production times, amounts and sequences;
- transportation flows;
- inventory levels and inventory deviations;
- sales and unmet demand amounts;
- product prices and price changes;

so as to maximize the total profit with the maintenance of the inventory levels and price levels.

Mathematical Formulation

The considered supply chain planning problem has been formulated as an MILP model, the details of which are described as follows.

Production sequences

The following constraints for the production sequences are adapted from the MILP model for the medium-term planning for multiproduct continuous plants proposed by Liu et al.[18,19] to model production in multiple plants.

At each plant, there is one product assigned to the first or last one to process in each time period

$$\sum_{i \in PS_n} F_{int} = 1, \qquad \forall n \in S, t \tag{1}$$

$$\sum_{i \in PS_n} L_{int} = 1, \qquad \forall n \in S, t \tag{2}$$

The aforementioned constraints are based on the assumption that at least one product is processed at each plant in each time period, which can be relaxed by introducing a pseudo product.[19]

For any product assigned to be processed at one plant in one time period, there is only one product assigned immediate before or after it, except for the first or last one

$$\sum_{i \in PS_n, i \neq j} Z_{ijnt} = E_{jnt} - F_{jnt}, \qquad \forall n \in S, j \in PS_n, t \tag{3}$$

$$\sum_{j \in PS_n, j \neq i} Z_{ijst} = E_{int} - L_{int}, \qquad \forall n \in S, i \in PS_n, t \tag{4}$$

If a product is the first or last one to be processed at one plant in a time period, there is exactly one changeover from the last product in the previous time period or to the first product in the next time period

$$\sum_{i \in PS_n} ZF_{ijnt} = F_{jnt}, \qquad \forall n \in S, j \in PS_n, t > 1 \tag{5}$$

$$\sum_{j \in PS_n} ZF_{ijnt} = L_{in,t-1}, \qquad \forall n \in S, i \in PS_n, t > 1 \tag{6}$$

If product i is processed precedent to product j at one plant in one time period, the order index of product j is higher than that of product i; otherwise if the product is not processed at one plant in one time period, its corresponding order index is zero

$$O_{jnt} - (O_{int} + 1) \geq -N \cdot (1 - Z_{ijnt}), \qquad \forall n \in S, i, j \in PS_n, i \neq j, t \tag{7}$$

$$O_{int} \leq N \cdot E_{int}, \qquad \forall n \in S, i \in PS_n, t \tag{8}$$

where N is the maximum number of products that one plant can process, i.e., $\max_{n \in S} |PS_n|$. It is worth mentioning that Eq. 7 can avoid the occurrences of subtours in the optimal production schedules.[18–20]

Production times

The production time of each product at one plant in each time period is limited between upper and lower bounds

$$\theta^L \cdot E_{int} \leq PT_{int} \leq \theta^U \cdot E_{int}, \qquad \forall n \in S, i \in PS_n, t \tag{9}$$

The changeover time between two consecutive time periods can be split into two parts in different time periods

$$CET1_{nt} + CET2_{n,t-1} = \sum_{i \in PS_n} \sum_{j \in PS_n} \tau^C_{ijn} \cdot ZF_{ijnt}, \qquad \forall s \in S, t > 1 \tag{10}$$

At each plant, the total production time plus the total changeover time should not exceed the total available time in each time period

$$\sum_{i \in PS_n} PT_{int} + \sum_{i \in PS_n} \sum_{j \in PS_n, j \neq i} \tau^C_{ijn} \cdot Z_{ijnt} + CET1_{nt}\big|_{t>1} + CET2_{nt}\big|_{t<t^L} \leq \theta^U, \forall n \in S, t \tag{11}$$

It needs to be mentioned that variables $CET1_{nt}$ and $CET2_{nt}$, and Eqs. 10 and 11 are adopted from literature on production planning and scheduling.[21]

Production amounts

The production amount of one product at one plant in each time period is equal to its production time multiplied by the corresponding processing rate

$$PR_{int} = r_{in} \cdot PT_{int}, \qquad \forall n \in S, i \in PS_n, t \tag{12}$$

Inventory

At each echelon node in the supply chain, the inventory level of one product in one time period is equal to its inventory in the previous time period, plus the total incoming flows from other connected echelon nodes and production amounts if the echelon node is a plant, minus the total outgoing flows to other connected echelon nodes and sales volume if the echelon node is a market, within the time period

$$I_{int} = I_{in,t-1}\big|_{t>1} + I^0_{in}\big|_{t=1} + PR_{int}\big|_{n \in S} + \sum_{n' \in T_{in}} Q_{in'n,t-\tau_{in'n}} - \sum_{n' \in \bar{T}_{in'}} Q_{inn't} - SV_{int}\big|_{n \in M}, \forall n, i \in PI_n, t \tag{13}$$

The inventory level of each product at one echelon node in each time period is limited between the upper and lower bounds

$$I^{\min}_{in} \leq I_{int} \leq I^{\max}_{in}, \qquad \forall n, i \in PI_n, t \tag{14}$$

Price elasticity of demand

Price elasticity is the concept that determines the relationship between product price and its demand, which is used to measure the degree of responsiveness of demand to change in price.[41] The price elasticities are almost always negative by the law of demand,[42] which means that a decrease in product price leads to increase in product demand, and vice versa, although the price elasticities may be positive in some special cases.[43] The price elasticity coefficient of each product at each market is defined as the division of percentage change in quantity of the product demanded by the percentage change in the price[44]

$$PE_{in} = \frac{\text{percentage change in demand}}{\text{percentage change in price}}$$

Based on the aforementioned equality, the relationship between the product price and its final demand is formulated as follows

$$\frac{D_{int} - D_{int}^0}{D_{int}^0} = PE_{in} \cdot \frac{P_{int} - P_{in}^0}{P_{in}^0}, \qquad \forall n \in M, i \in PM_n, t \quad (15)$$

where the initial demand D_{int}^0 is uncertain. It follows a uniform distribution between $(1 - \alpha_{int}) \cdot DF_{int}^0$ and $(1 + \alpha_{int}) \cdot DF_{int}^0$, where the DF_{int}^0 is the expected value of D_{int}^0, as well as the forecast of initial demands, and $\alpha_{int} \in (0,1)$ is the forecast error of D_{int}^0. When the initial demand D_{int}^0, initial price, P_{in}^0 and price elasticity PE_{in} are known, the final demand D_{int} is determined after the pricing decision P_{int}, is made.

Unmet demands

The unmet demand amount is equal to the demand minus the sales volume of each product at each market in each time period

$$U_{int} = D_{int} - SV_{int}, \qquad \forall n \in M, i \in PM_n, t \quad (16)$$

Pricing

Only one available price level should be selected for each product at each market in each time period

$$\sum_k Y_{intk} = 1, \qquad \forall n \in M, i \in PM_n, t \quad (17)$$

$$P_{int} = \sum_k \overline{P}_{ink} \cdot Y_{\text{int}k}, \qquad \forall n \in M, i \in PM_n, t \quad (18)$$

Inventory deviation

The inventory deviation from the corresponding inventory trajectory at each echelon node is the absolute value of the difference between the inventory and inventory trajectory

$$I_{int}^D = \left| I_{int}^T - I_{int} \right|, \qquad \forall n, i \in PI_n, t \quad (19)$$

Here, we use the L_1 rather than L_2 norm to maintain model linearity and to avoid any overemphasis on outlier values of inventory which are not patently damaging to the system (contrast to process control applications).

As the absolve value function in the aforementioned constraint is nonlinear, we rewrite it using two linear inequalities. As the inventory deviation is minimized in the objective function, which will be introduced later, Eq. 19 can be rewritten as Eqs. 20 and 21

$$I_{int}^D \geq I_{int}^T - I_{int}, \qquad \forall n, i \in PI_n, t \quad (20)$$

$$I_{int}^D \geq I_{int} - I_{int}^T, \qquad \forall n, i \in PI_n, t \quad (21)$$

Price change

In order to keep the price fluctuation at a low level, we need to control the price change. The price change of each product at each market can be defined as the absolute difference between the prices in two consecutive time periods, which is given as

$$PC_{int} = \left| P_{int} - P_{in,t-1} \right|, \qquad \forall n \in M, i \in PM_n, t \quad (22)$$

where $P_{in0} = P_{in}^0$. Similar to the inventory deviation, Eq. 22 can be rewritten as the following two inequalities, Eqs. 23 and 24

$$PC_{int} \geq P_{int} - P_{in,t-1}\big|_{t>1} - P_{in}^0\big|_{t=1}, \qquad \forall n \in M, i \in PM_n, t \quad (23)$$

$$PC_{int} \geq P_{in,t-1}\big|_{t>1} + P_{in}^0\big|_{t=1} - P_{int}, \qquad \forall n \in M, i \in PM_n, t \quad (24)$$

It should be added that an alternative pricing strategy can consider the price change from the initial prices instead of the previous price.

Profit

The total profit is calculated by the sales revenue, production cost, changeover cost, transportation cost, and unmet demand cost

$$\Phi_1 = TotR - TotPC - TotCC - TotTC - TotUC \quad (25)$$

It is worth noting that the total inventory cost is not considered in the profit to avoid the confliction between the inventory control and profit maximization.

The total revenue is the sum of the sales volume of all products multiplied by the corresponding prices

$$TotR = \sum_t \sum_{n \in M} \sum_{i \in PM_n} SV_{int} \cdot P_{int} \quad (26)$$

Incorporating with Eq. 18, Eq. 26 can be rewritten as

$$TotR = \sum_t \sum_{n \in M} \sum_{i \in PM_n} \sum_k \overline{P}_{ink} \cdot SV_{int} \cdot Y_{\text{int}k} \quad (27)$$

In Eq. 27, the nonlinear term $SV_{int} \cdot Y_{intk}$ can be substituted by the introduced auxiliary positive variable SY_{intk} to substitute the nonlinear term in the objective function and the two more constraints to enforce $SY_{\text{int}k} \equiv SV_{int} \cdot Y_{\text{int}k}$

$$SY_{intk} \leq N \cdot Y_{intk}, \qquad \forall n \in M, i \in PM_n, t, k \quad (28)$$

$$SV_{int} = \sum_k SY_{\text{int}k}, \qquad \forall n \in M, i \in PM_n, t \quad (29)$$

where N is a large number, can be the upper bound of the sales in time period t. So the following constraint is equivalent to Eq. 27

$$TotR = \sum_t \sum_{n \in M} \sum_{i \in PM_n} \sum_k \overline{P}_{ink} \cdot SY_{\text{int}k} \quad (30)$$

The total production cost is calculated by the production amount multiplied by the corresponding production cost

$$TotPC = \sum_t \sum_{n \in S} \sum_{i \in PS_n} CP_{in} \cdot PR_{int} \quad (31)$$

The total changeover cost is the summation of the costs of all occurred changeovers

Table 1. Suitability of all Echelon Nodes

		Products									
		I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
Plants	S1	√	√	√	√	√	√				
	S2			√	√	√	√	√	√		
	S3					√	√	√	√	√	√
DCs	C1	√	√	√	√						
	C2	√	√	√	√						
	C3			√	√	√	√				
	C4			√	√	√	√				
	C5					√	√	√	√		
	C6					√	√	√	√		
	C7							√	√	√	√
	C8							√	√	√	√
Markets	M1	√	√	√	√						
	M2	√	√	√	√						
	M3	√	√	√	√						
	M4	√	√	√	√						
	M5			√	√	√	√				
	M6			√	√	√	√				
	M7			√	√	√	√				
	M8			√	√	√	√				
	M9					√	√	√	√		
	M10					√	√	√	√		
	M11					√	√	√	√		
	M12					√	√	√	√		
	M13							√	√	√	√
	M14							√	√	√	√
	M15							√	√	√	√
	M16							√	√	√	√

$$TotCC = \sum_{t}\sum_{s\in S}\sum_{i\in PS_n}\sum_{j\in PS_n, j\neq i} CC_{ijn}\cdot Z_{ijnt} + \sum_{t>1}\sum_{s\in S}\sum_{i\in PS_n}\sum_{j\in PS_n} CC_{ijn}\cdot ZF_{ijnt} \quad (32)$$

The total transportation cost is the summation of the transportation costs from plants to distribution centers and from distribution centers to markets, which is equal to the unit transportation cost multiplied by the product flows

$$TotTC = \sum_{t}\sum_{n}\sum_{n'\in \bar{T}_{in}}\sum_{i\in PI_n} CT_{inn'}\cdot Q_{inn't} \quad (33)$$

The total unmet demand cost is determined by the unmet demand amount and the unit unmet demand cost

$$TotUC = \sum_{t}\sum_{n\in M}\sum_{i\in PM_n} CU_{in}\cdot U_{int} \quad (34)$$

Weighed total inventory deviation

The weighted total inventory deviation is the summation of the total inventory deviation in each echelon multiplied by the corresponding weight

$$\Phi_2 = w^I \cdot \sum_{t}\sum_{n}\sum_{i\in PI_n} I^D_{int} \quad (35)$$

Weighed total price change

The weighted total price change is the summation of the total price change multiplied by the corresponding weight

$$\Phi_3 = w^P \cdot \sum_{t}\sum_{n\in M}\sum_{i\in PM_n} PC_{int} \quad (36)$$

Objective function

The objective is to maximize the profit with the maintenance of the inventory levels and price levels. So, the profit is penalized by the weighed inventory deviation and price changes are in the objective function

$$\Pi = \Phi_1 - \Phi_2 - \Phi_3 \quad (37)$$

Overall, the production, distribution and inventory planning problem has been formulated as an MILP model with Eqs. 1–18, 20, 21, 23–25, and 28–36 as the constraints and Eq. 37 as the objective function.

MPC Approach

MPC, also referred to as model-based predictive control, receding horizon control or moving horizon optimal control,[45] is an advanced method of process control widely used in the process industry for over 30 years.[46–51] It has been investigated in the literature and successfully applied to supply chains during the last decade.[27,52–59] The review articles by Ortega and Lin[60] and Sarimveis et al.[61] presented an overview of the application of control theory, including MPC on supply chains.

Here, the MPC approach is used to implement the proposed aforementioned MILP model. The main idea of MPC is to choose the control action by repeatedly solving online an optimal control problem, aiming to optimize a performance criterion, which consists of the deviation of the future controlled process from a reference trajectory over a future horizon.[27,61] In our MPC approach, the disturbance is the

Table 2. Sequence-Dependent Changeover Times (Hours)

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
I1	0	2.25	3	4	2.5	3	–[a]	–	–	–
I2	3.5	0	1.5	2.75	1.5	2	–	–	–	–
I3	3	2.25	0	3.5	3.25	2.5	3.5	4	–	–
I4	3.5	3	3.25	0	5	1.5	3	2.5	–	–
I5	2.5	3.5	3	4.5	0	4	2.75	3.25	4	4
I6	4.5	4	3.5	3.25	2.75	0	4	1.5	4.5	5
I7	–	–	3	4	2	2.25	0	2.5	2.5	3
I8	–	–	1.5	3	2.75	5	3.5	0	3.5	4
I9	–	–	–	–	4	3.5	3	3.25	0	4
I10	–	–	–	–	4.5	5	1.5	2	5	0

[a]Impossible production changeovers

initial demand D^0_{int}. In each time period, the initial demands in the current time period, t^*, are realized, while all the future demands in the control horizon $T^C = [t^*, t^* + L^{CH} - 1]$ are unknown. So, initial demand forecast DF^0_{int}, are used in the future time periods $t^* < t < t^* + L^{CH}$. In this case, Eq. 15 can be rewritten as

$$\frac{D_{int} - D^0_{int}}{D^0_{int}} = PE_{in} \cdot \frac{P_{int} - P^0_{in}}{P^0_{in}}, \qquad \forall n \in M, i \in PM_n, t = t* \tag{38}$$

$$\frac{D_{int} - DF^0_{int}}{DF^0_{int}} = PE_{in} \cdot \frac{P_{int} - P^0_{in}}{P^0_{in}}, \qquad \forall n \in M, i \in PM_n, t \in T^C \backslash \{t*\} \tag{39}$$

The optimization MILP model for the control horizon T^c, is described as follows

$$\max \Pi = \Phi_1 - \Phi_2 - \Phi_3 \tag{OP1}$$

s.t. Eqs. 1–14, 16–18, 20, 21, 23–25, 28–36, 38 and 39 specific for $t \in T^C$

In each current time period t^*, we solve the optimization problem OP1, and then fix the obtained operating decisions for time period t^*. Then, we move to the next time period and repeat the aforementioned steps until all the time periods have been considered. The MPC approach implemented for this supply chain planning problem is described as follows:

STEP 1: Initialize the current time period $t^* = 1$;

STEP 2: Update the control horizon $T^C = [t^*, t^* + L^{CH} - 1]$;

STEP 3: Generate the initial demand for the current time period t^*,

$$D^0_{int^*} = \text{Uniform}[(1 - \alpha_{int^*}) \cdot DF^0_{int^*}, (1 + \alpha_{int*}) \cdot DF^0_{int^*}], \quad \forall n \in M, i \in PM_n$$

STEP 4: Solve the MILP model OP1 for the control horizon;

STEP 5: Fix the values of the all variables for the current time period t^*;

STEP 6: If $t^* = t^L$, STOP; Otherwise, let $t^* = t^*+1$, go to STEP 2.

A Numerical Example

The supply chain example considered here has three echelons with three plants (S1–S3), eight distribution centers (C1–C8), 16 markets (M1–M16). See Figure 1 for the structure of the supply chain. There are 10 products (I1–I10) in the supply chain. Table 1 shows the suitability of all echelon nodes, including plants, DCs, and markets. We consider a planning horizon of 1 year, which is divided into 52 weeks. The minimum and maximum production times in each week are 5 and 168 h, respectively. We assume that the sequence-dependent changeover times and costs between two products occurring in different plants are the same. The changeover times in hours are presented in Table 2. The unit changeover cost is 60 k$/h. Thus, the cost of each changeover in k$ is equal to the corresponding changeover time in hours multiplied by 60, e.g., the changeover cost from I1 to I2 is 60 × 2.25 = 135 k$. The production rates and unit production costs at suitable plants are given in Tables 3 and 4, respectively.

The transportation times between different echelons are given in Tables 5 and 6. The unit cost of transportation for 1 week is 1 k$/ton. Thus, the transportation costs in k$/ton are equal to the corresponding transportation times in weeks, e.g., the unit transportation cost from plant S1 to DC C4 is 2 k$/ton.

The product initial demand in each week at each market is uncertain and follows a uniform distribution between the known specific upper and lower bounds. Before the initial demand realization, their forecasts are known, which are expected values of actual demands, and used in the optimization problem of MPC to predict future outputs. The total initial demand forecast is 69,460 ton. In each market, the maximum initial demand forecast for one product in one time period is 40 ton, while the minimum initial demand forecast is 5 ton. The forecast error α_{int}, varies among different products and markets, and the maximum value is 20%.

For each product, the inventory trajectories at markets in each week are set to 2 times of its maximum initial demand forecast at the market; the inventory trajectories at DCs are set to 4 times of maximum initial demand forecast of the product at one market; the inventory trajectories at markets are set to 8 times of maximum initial demand forecast of the product at one market. The inventory trajectories at the

Table 3. Production Rates (ton/h)

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
S1	3.5	3	3.5	4	3.5	4				
S2			4	4.5	3	3.5	4.5	5		
S3					3.5	4	3.5	5.5	3.5	4

Table 4. Unit Production Costs (k$/ton)

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
S1	2	1	3	2	2.5	5				
S2			1	2.5	3	4	2	3		
S3					2	4.5	2	3.5	1	2

Table 5. Transportation Times from Plants to DCs (Weeks)

	C1	C2	C3	C4	C5	C6	C7	C8
S1	1	1	1	2	–[a]	–	–	–
S2	–	–	1	2	1	2	–	–
S3	–	–	–	–	1	1	2	1

[a]Connections between plants and DCs are not allowed

Table 6. Transportation Times from DCs to Markets (Weeks)

	M1	M2	M3	M4	M5	M6	M7	M8
C1	1	1	0	1	–[a]	–	–	–
C2	0	1	1	1	–	–	–	–
C3	–	–	–	–	0	1	1	1
C4	–	–	–	–	1	1	1	0
	M9	M10	M11	M12	M13	M14	M15	M16
C5	1	1	0	1	–	–	–	–
C6	1	1	1	0	–	–	–	–
C7	–	–	–	–	1	1	1	0
C8	–	–	–	–	0	1	1	1

[a]Connections between DCs and markets are not allowed

suitable echelon nodes are given in Table 7. It is assumed that the initial inventories of each echelon node at the beginning of the planning horizon are the same as the corresponding inventory trajectories, i.e., $I_{in}^0 = I_{in0}^T$ in order to avoid the inventory deviations at the beginning of the planning horizon.

The actual demands are determined by the price elasticity, initial demands and selected price levels obtained from the optimization problem in MPC. The price elasticity coefficient for each product in each market is given in Table 8. Table 9 shows the available price levels (K1–K5) for selection, in which the prices at level K3 (in bold) are the initial prices. The unit unmet demand cost of each product is assumed to be half of its initial price at the market.

Results and Discussion

The proposed MPC approach is implemented in GAMS 22.8[62] using MILP solver CPLEX 11.1[63] in a Windows XP environment on a Pentium 4 3.40 GHz, 1.00 GB RAM machine. For the supply chain example discussed, there are 52 MILP subproblems in total to be solved iteratively for the implementation of the MPC approach. The optimality gap for each MILP model is set to 5%. The CPU time limit of each MILP model is 3,600 s.

Unless indicated specifically, in the objective function, the weight for the inventory deviations is set to 2.5, and the weight for the price change is set to 10, i.e., $w^I = 2.5$ and $w^P = 10$.

In this section, we will investigate the computational results of the example by the proposed MILP-based MPC approach and discuss the effects of the length of the control horizon, price elasticity of demand, weights, inventory trajectories, and changeovers on the solutions.

Length of the control horizon

In order to find the optimal length of the control horizon for the MPC approach, we consider three approaches with different lengths of control horizon, which are 4, 5 and 6 weeks.

The computational results for all three approaches are presented in Table 10. The approach with $L^{CH} = 4$ has the worst performance among all the three approaches, and its objective value is only 70% of those of the other two approaches, which results from the much higher inventory deviation. The approaches with $L^{CH} = 5$ and $L^{CH} = 6$ have similar objective values, profits, inventory deviations and price changes. As the approach with a longer control horizon takes much more CPU time, the approach with $L^{CH} = 5$ spends only about 1/4 of CPU time of the approach $L^{CH} = 6$. From the aforementioned results, we can see that a shorter length of control horizon has an advantage in the

Table 7. Inventory Trajectories at all Echelon Nodes (Ton)

		I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
Plant	S1	160	160	240	160	320	320				
	S2			240	160	320	320	320	160		
	S3					320	320	320	160	240	240
DC	C1	80	80	120	80						
	C2	80	80	120	80						
	C3			120	80	160	160				
	C4			120	80	160	160				
	C5					160	160	160	80		
	C6					160	160	160	80		
	C7							160	80	120	120
	C8							160	80	120	120
Market	M1	40	40	60	40						
	M2	40	40	60	40						
	M3	40	40	60	40						
	M4	40	40	58	40						
	M5			60	40	80	80				
	M6			58	40	80	80				
	M7			58	38	78	80				
	M8			60	40	80	80				
	M9					80	80	80	38		
	M10					80	80	78	40		
	M11					80	80	78	40		
	M12					80	78	78	40		
	M13							78	40	60	60
	M14							78	40	60	58
	M15							80	38	60	60
	M16							80	40	58	58

Table 8. Price Elasticity Coefficients

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
M1	−1.68	−1.04	−1.44	−1.36						
M2	−1.76	−1.12	−1.44	−1.44						
M3	−1.60	−1.28	−1.36	−1.28						
M4	−1.84	−1.20	−1.36	−1.36						
M5			−1.44	−1.52	−1.04	−1.36				
M6			−1.28	−1.36	−1.12	−1.44				
M7			−1.44	−1.44	−1.12	−1.44				
M8			−1.28	−1.36	−1.04	−1.36				
M9					−0.96	−1.44	−1.76	−1.20		
M10					−0.96	−1.44	−2.00	−1.28		
M11					−1.04	−1.52	−1.68	−1.28		
M12					−0.88	−1.44	−1.60	−1.20		
M13							−1.92	−1.36	−2.08	−1.44
M14							−1.76	−1.36	−2.32	−1.52
M15							−1.84	−1.28	−2.08	−1.44
M16							−1.68	−1.36	−2.00	−1.36

computational time, due to the less complexity of each MILP subproblem in the MPC approach, while a longer length of control horizon has better computational results because of more future demand patterns foreseen in each MILP subproblem. In this work, considering the trade off between the computational time and accuracy, we use the approach with $L^{CH} = 5$ for the further computation and discussion.

Moreover, in all three cases, the total actual final demand is less than the total initial demand, which is 69,260 ton, which implies the average selected prices are higher than the initial prices. It is worth noting that when a longer control horizon is used, a higher actual final demand after pricing is realized, implying that lower product price levels are selected at the markets.

Effect of price elasticity of demand

We investigate the effect of the price elasticity of demand by comparing the following two pricing strategies (PS1 and PS2):

PS1: Pricing with price elasticity of demand, where the price elasticity coefficients are as given in Table 8, and the price change defined as Eqs. 23 and 24 are included in the optimization model;

Table 9. Available Price Levels (k$/Ton)

		Price levels							Price levels				
		K1	K2	**K3**	K4	K5			K1	K2	**K3**	K4	K5
I1	M1	8	9.5	**11**	12.5	14	I6	M5	7	8.5	**10**	11.5	13
	M2	6	7.5	**9**	10.5	12		M6	8	9.5	**11**	12.5	14
	M3	7	8.5	**10**	11.5	13		M7	9	10.5	**12**	13.5	15
	M4	6	7.5	**9**	10.5	12		M8	8	9.5	**11**	12.5	14
I2	M1	4	5	**6**	7	8		M9	7	8.5	**10**	11.5	13
	M2	3	4	**5**	6	7		M10	8	9.5	**11**	12.5	14
	M3	4	5	**6**	7	8		M11	7	8.5	**10**	11.5	13
	M4	3	4	**5**	6	7		M12	7	8.5	**10**	11.5	13
I3	M1	8	9.5	**11**	12.5	14	I7	M9	6	7.5	**9**	10.5	12
	M2	7	8.5	**10**	11.5	13		M10	5	6.5	**8**	9.5	11
	M3	9	10.5	**12**	13.5	15		M11	6	7.5	**9**	10.5	12
	M4	8	9.5	**11**	12.5	14		M12	5	6.5	**8**	9.5	11
	M5	7	8.5	**10**	11.5	13		M13	5	6.5	**8**	9.5	11
	M6	8	9.5	**11**	12.5	14		M14	5	6.5	**8**	9.5	11
	M7	9	10.5	**12**	13.5	15		M15	6	7.5	**9**	10.5	12
	M8	7	8.5	**10**	11.5	13		M16	5	6.5	**8**	9.5	11
I4	M1	10	12	**14**	16	18	I8	M9	8	9.5	**11**	12.5	14
	M2	9	11	**13**	15	17		M10	7	8.5	**10**	11.5	13
	M3	8	10	**12**	14	16		M11	8	9.5	**11**	12.5	14
	M4	9	11	**13**	15	17		M12	8	9.5	**11**	12.5	14
	M5	8	10	**12**	14	16		M13	8	9.5	**11**	12.5	14
	M6	9	11	**13**	15	17		M14	9	10.5	**12**	13.5	15
	M7	8	10	**12**	14	16		M15	8	9.5	**11**	12.5	14
	M8	8	10	**12**	14	16		M16	8	9.5	**11**	12.5	14
I5	M5	6	7.5	**9**	10.5	12	I9	M13	4	5	**6**	7	8
	M6	5	6.5	**8**	9.5	11		M14	4	5	**6**	7	8
	M7	4	5.5	**7**	8.5	10		M15	3	4	**5**	6	7
	M8	6	7.5	**9**	10.5	12		M16	3	4	**5**	6	7
	M9	5	6.5	**8**	9.5	11	I10	M13	13	15	**17**	19	21
	M10	6	7.5	**9**	10.5	12		M14	12	14	**16**	18	20
	M11	6	7.5	**9**	10.5	12		M15	13	15	**17**	19	21
	M12	4	5.5	**7**	8.5	10		M16	14	16	**18**	20	22

Table 10. Comparison Among the Three Cases with Different Control Horizon Lengths

		$L^{CH} = 4$	$L^{CH} = 5$	$L^{CH} = 6$
Objective		210,220	295,915	293,379
Profit (k$)		348,511	340,379	337,864
Revenue (k$)		623,759	640,785	645,883
Production cost (k$)		134,476	145,227	149,225
Changeover cost (k$)		45,045	49,230	43,725
Transportation cost (k$)		95,585	105,778	114,836
Unmet demand cost (k$)		142	171	233
Inventory deviation (ton)	Plant	273	1,319	2,641
	DC	22,715	7,206	5,130
	Market	30,972	7,668	8,245
Price change (k$/ton)		339	398	445
Actual final demand (ton)		56,394	58,911	59,872
CPU (s)		723	2,045	7,953

PS2: Pricing without price elasticity of demand, where the price elasticity coefficients $PE_{in} = 0$, and prices and demands are fixed to their initial values, i.e., $P_{in} = P^0_{in}$ and $D_{int} = D^0_{int}$, $\forall n \in M$, $i \in PM_n$, t.

The solutions of the aforementioned two pricing strategies obtained by MPC are given in Table 11. Due to the lack of flexibility for pricing, PS2 generates a lower objective value, with lower profit and higher inventory deviation, than PS1. Meanwhile, the total actual final demand under PS1 is lower than that under PS2 (58,911 ton vs. 69,260 ton), due to the price elastic of demand and the higher price levels of PS1 than the fixed price levels of PS2. Figure 2 presents the average price of each product under pricing strategy PS1, from which we can conclude that the proposed approach with price elasticity of demand has more flexibility of price and demand control and performs well to reduce the risk of the supply chain brought by the great price fluctuation.

Effect of weights

Here, we examine the effect of values of weights for inventory deviations and price change on the profit and total inventory deviation. The profit is expressed by Φ_1 in Eq. 25, while the total inventory deviation is expressed by Φ'_2 as follows:

$$\Phi'_2 = \sum_t \sum_n \sum_{i \in PI_n} I^D_{int} \tag{40}$$

Here, we consider the value of w^I varies from 1 to 3 by a step length of 0.5, and the value of w^P is equal to 10 and 50. The fixed pricing strategy, where the prices are fixed to the initial prices, are also investigated, which can be considered as a special case with a very large value of w^P. In Figure 3, different values of w^P generate different curves. On each curve, the left end node represents the case with the largest value of, i.e., $w^I = 3$, while the right end node represents the case with the smallest value of, i.e., $w^I = 1$. The other points on each curve in Figure 3 represent the solutions using the values of w^I between 1 and 3, which decrease from left to right. For a fixed value of w^P, with a higher penalty on inventory deviation w^I, the inventory deviation decrease. In order to maintain a more stable inventory level, the whole supply chain earns less profit. So a higher value of w^I has a negative effect on profit and a positive effect on inventory maintenance, as shown in Figure 3. When the value of w^I is fixed, a higher value of w^P can lead to a lower profit and a larger inventory deviation, as the less flexibility on pricing impacts the supply chain performance.

Table 11. Comparison between the Two Pricing Strategies with and without Price Elasticity of Demand

		PS1	PS2
Objective		265,889	295,915
Profit (k$)		318,744	340,379
Revenue (k$)		675,522	640,784
Production cost (k$)		172,310	145,227
Changeover cost (k$)		59,160	49,230
Transportation cost (k$)		122,501	105,778
Unmet demand cost (k$)		2,807	171
Inventory deviation (ton)	Plant	1,247	1,319
	DC	10,281	7,206
	Market	9,614	7,668
Price change (k$/ton)		0	398
Actual final demand (ton)		69,260	58,911

Effect of inventory trajectories

Figure 4 shows the average inventory deviations in percentage at all three echelons with price elasticity of demand. We can see that the average inventory deviations at all the echelons are very small. The inventory deviation at the plants is the closest to zero, within 4% in all the weeks. At the markets, the average inventory deviations are within 4%, apart from the first 3 weeks. The average inventory deviations at the DC are the highest, but still within 10% except the 2 weeks. Overall, the inventories at all echelons are maintained at stable levels, and the inventory fluctuation is not significant. It is worth mentioning that the total production in this case is 60,381 ton, which is slightly more than the total sales, 58,845 ton. Meanwhile, the lost sales level is only 66 ton, and the total initial inventory and final inventories are 12,128 ton and 12,085 ton, respectively. Thus, the production does not only try to cover most of the demands, but also maintain the inventory levels.

Here, we reduce the inventory trajectories, as well as the initial inventory, to be zero, to examine the effect of the inventory trajectories on the solutions. Thurs, we compare the following two scenarios (SC1 and SC2):

SC1: With the inventory trajectories given in Table 7;

SC2: With the inventory trajectories equal to zero at all echelon nodes.

As SC2 has zero initial inventory level; it has to reduce the actual final demand by increasing the price levels (Figure 5) to satisfy the demand as much as possible. However, there are still quite high unmet demands in SC2 (2,635 ton)

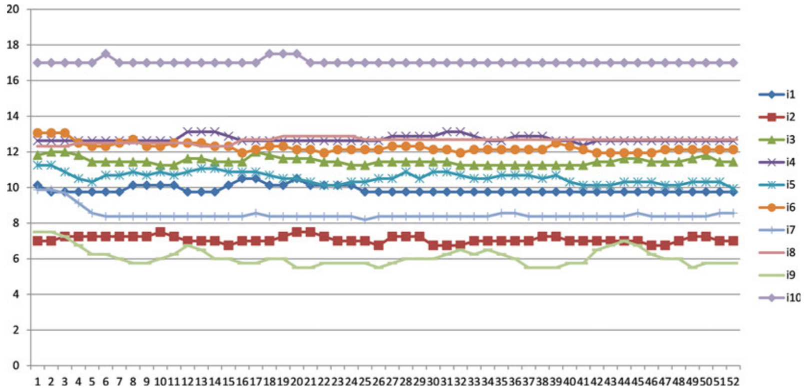

Figure 2. Average price of each product over all markets under pricing strategy PS1.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

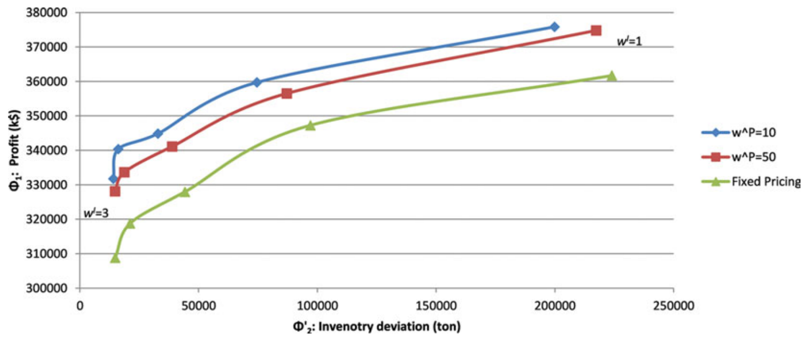

Figure 3. Effect of weights on profit and inventory deviation.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

compared to the total final demand (57,924 ton), while SC1 generates unmet demands of 66 ton out of total final demand of 58,911 ton. Thus, much less sales occur in SC2 with zero inventory trajectories, which lead to over 10% lower objective and profit than SC1 (Table 12). Comparing the deviation and price change, Table 12 shows SC2 produces less inventory deviations and more fluctuant prices. We can conclude that the effect of lower inventory trajectories on the supply chain performance is significant, which result in higher unmet demands and lower profit. Thus, inventory trajectory levels should be kept high enough to avoid the unmet demands as low as possible.

Effect of changeovers

The sequence-dependent changeovers are considered in the proposed MILP model. Although the constraints for the sequence-dependent changeovers in the MILP model are heavy and increase the computational complexity of the proposed model, the necessity for considering changeovers in the MILP model is to be verified later.

In order to examine whether the sequence-dependent changeover is crucial to be considered simultaneously with the other constraints in the proposed MILP at the cost of the computational time, we propose a hierarchical approach as

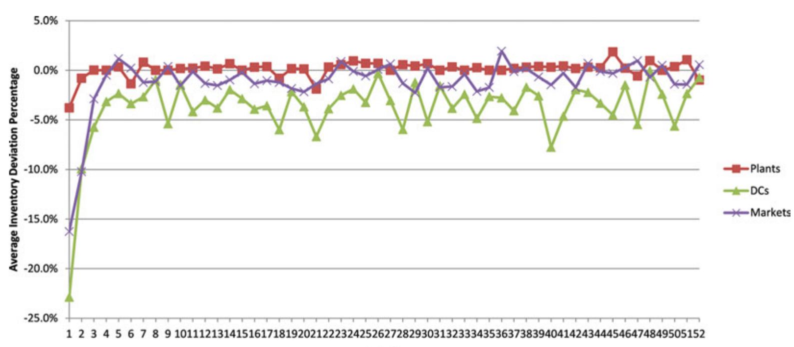

Figure 4. Average inventory deviation in each echelon.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

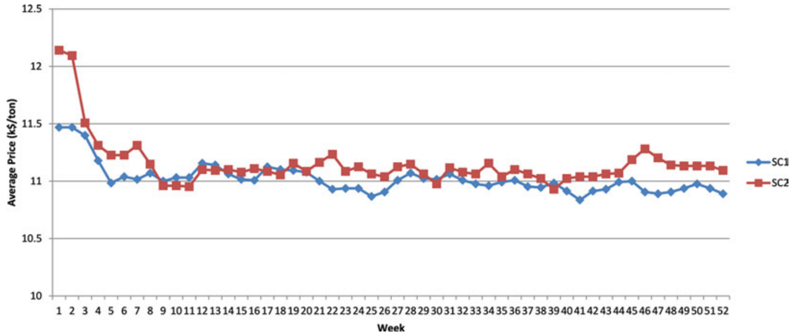

Figure 5. Average prices in scenario SC1 and SC2.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

another way to tackle the sequence-dependent changeovers. In the hierarchical approach, we first solve a simpler MILP model which does not consider the changeovers. Its differences from the proposed original MILP model are as follows:

1. The terms for changeover costs are not included in the objective function;
2. Eqs. 1–8 and 10 are omitted as the constraints;
3. The Eq. 11 is replaced by the following constraint

$$\sum_{i \in PS_n} PT_{int} \leq \theta^U, \qquad \forall n \in S, t \tag{41}$$

In the MPC, the MILP model (OP2) in the hierarchical approach is given by

$$\max \Pi = TotR - TotPC - TotTC - TotUC - \Phi_2 - \Phi_3 \tag{OP2}$$

s.t. Eqs. 9, 12–18, 20, 21, 23, 24, 28–36, 38, 39 and 41 specific for $t \in T^C$

Then, the production allocations obtained from OP2 are fixed, and the optimal production sequence is determined by minimizing the total changeover time in the following MILP model (OP3)

$$\min \sum_{t \in T^C} \sum_{n \in S} \sum_{i \in PS_n} \sum_{j \in PS_n, j \neq i} \tau_{ijn} \cdot Z_{ijnt} + \sum_{t \in T^C \setminus \{1\}} \sum_{n \in S} \sum_{i \in PS_n} \sum_{j \in PS_n} \tau_{ijn} \cdot ZF_{ijnt} \tag{OP3}$$

s.t. Eqs. 1–8, 10 specific for $t \in T^C$

Finally, the obtained production allocations and sequences are fixed before solving the reduced original MILP model OP1.

Overall, we use the following steps to replace STEP 4 in the MPC algorithm:

STEP 4.1: Solve the MILP model OP2 without production sequences for the control horizon;
STEP 4.2: Fix the binary variable E_{int} for the control horizon;
STEP 4.3: Solve the MILP model OP3 to minimize changeover times;
STEP 4.4: Fix the binary variables: F_{int}, L_{int}, Z_{ijnt}, and ZF_{ijnt} for the control horizon;
STEP 4.5: Solve the reduced MILP model OP1 for the control horizon;
STEP 4.6: Free all binary variables E_{int}, F_{int}, L_{int}, Z_{ijnt} and ZF_{ijnt} for the future time periods in the control horizon $t^* < t < t^* + L^{CH}$

From the comparison in Table 13, the CPU time of the hierarchical approach is much faster than the original MILP model as the same as expected, and the inventory deviations are lower in all three echelons. However, the optimal objective value and profit obtained from the hierarchical approach are both around 20% lower, and the price change is about 5 times higher, compared with the single-level MILP model. Also, there is much more unmet demand from the hierarchical approach. It is also worth noting that the total changeover cost in the optimal solution of the hierarchical approach is almost doubled. It is obvious that the hierarchical approach generates much more changeovers, which result in much less profit and objective value. Thus, it is proved that the consideration of the sequence-dependent changeovers

Table 12. Comparison between the Two Scenarios with Different Inventory Trajectories

		SC1	SC2
Objective		295,915	265,793
Profit (k$)		340,379	294,937
Revenue (k$)		640,785	604,887
Production cost (k$)		145,227	140,424
Changeover cost (k$)		49,230	52,185
Transportation cost (k$)		105,778	104,462
Unmet demand cost (k$)		171	12,879
Inventory deviation (ton)	Plant	1,319	1,230
	DC	7,206	1,598
	Market	7,668	5,809
Price change (k$/ton)		398	755
Actual final demand (ton)		58,911	57,924

Table 13. Comparison between the MILP Model and Hierarchical Approach

		MILP	Hierarchical
Objective		295,915	241,117
Profit (k$)		340,379	283,029
Revenue (k$)		640,785	629,021
Production cost (k$)		145,227	146,202
Changeover cost (k$)		49,230	92,220
Transportation cost (k$)		105,778	106,988
Unmet demand cost (k$)		171	581
Inventory deviation (ton)	Plant	1,319	111
	DC	7,206	2,333
	Market	7,668	5,646
Price change (k$/ton)		398	2,169
CPU (s)		2,045	161

simultaneously with other constraints in the MILP model is necessary, in despite of at the cost of computational complexity.

Concluding Remarks

In this work, we have addressed a multiechelon, multiproduct supply chain planning problem considering inventory and price maintenance, price elasticity of demand and sequence-dependent changeovers under demand uncertainty. An MILP model has been proposed with an objective function including the profit, inventory deviations from the trajectories, and price changes. To implement the proposed model, an iterative MPC approach has been adopted.

The proposed MPC approach has been applied to a supply chain example for discussion. Among three lengths of control horizon, $L^{CH} = 5$ was selected as the optimal one considering both objective value and CPU time. It is worth mentioning that the optimal length is dependent on the example discussed. Comparing the cases with and without price elasticity of demand, the pricing strategy with price elasticity of demand has a higher flexibility on price and demand management, which earns more profit and less inventory deviations than the pricing strategy with constant prices. Considering the effect of weights on both the profit and inventory deviation by examine five values of weights for inventory deviation and two values of weight for price change, the results show that the increased weights on inventory deviation and on price change both have a negative effect on the profit, while both of them have opposite effects on the inventory deviation.

With the given inventory trajectories, the inventory deviations at all three echelon of the supply chain are small. Comparing with the scenario with zero inventory trajectories, the results show that the levels of inventory trajectories have significant effect on the supply chain performance, and the inventory trajectories should be kept high enough to minimize the unmet demands. Moreover, the importance of changeover constraints in the proposed MILP model is verified after comparison with a hierarchical approach.

A future development this work could be the investigation of the optimal inventory trajectory levels, as well as the optimal initial inventory levels, in the optimization model. Another future research direction could be the incorporation of dynamic demand forecast, in which the future demand forecast is updated weekly.

Acknowledgments

S. Liu gratefully acknowledges the financial support from the Overseas Research Students Award Scheme (ORSAS) and the Centre for Process Systems Engineering (CPSE).

Notation

Indices

i, j = product
k = price level
n, n' = echelon node
t = time period
t^* = current time period in the control horizon
t^L = last time period in the planning horizon

Sets

M = set of markets
PI_n = set of products stored at echelon node n
PM_n = set of products sold at echelon node n
PS_n = set of products produced at echelon node n
S = set of plants
T_{in} = set of echelon nodes sending product i to echelon node n
$\overline{T}_{in}$ = set of echelon nodes receiving product i from echelon node n
T^C = set of time periods in the control horizon

Parameters

CC_{ijn} = unit changeover cost from product i to j produced at echelon node n
CP_{in} = unit production cost of product i produced at echelon node n
$CT_{inn'}$ = unit transportation cost of product i from echelon node n to n'
CU_{in} = unit unmet demand cost of product i sold at echelon node n
D^0_{int} = initial demand of product i sold at echelon node n in time period t
DF^0_{int} = initial demand forecast of product i sold at echelon node n in time period t
I^0_{in} = initial inventory of product i at echelon node n
$I^{\max}_{in}$ = maximum inventory of product i allowed at echelon node n
$I^{\min}_{in}$ = minimum inventory of product i allowed at echelon node n
I^T_{int} = inventory trajectory of product i at echelon node n in time period t
L^{CH} = length of the control horizon
N = a large number
P^0_{in} = initial price of product i sold at echelon node n
$\overline{P}_{ink}$ = price at level k of product i sold at echelon node n
PE_{in} = price elasticity coefficient of product i sold at echelon node n
r_{in} = processing rate of product i produced in echelon node n
w^I = control weight for inventory deviation
w^P = control weight for price change
α_{int} = forecast error of initial demand of product i sold at echelon node n in time period t
θ^L = lower bound for processing time in a time period
θ^U = upper bound for processing time in a time period
τ^C_{ijn} = changeover time from product i to product j produced at echelon node n
$\tau_{inn'}$ = transportation time of product i from echelon node n to n'

Binary variables

E_{int} = 1 if product i is produced at echelon node n in time period t; 0 otherwise
F_{int} = 1 if product i is the first one produced at echelon node n in time period t; 0 otherwise
L_{int} = 1 if product i is the last one produced at echelon node n in time period t; 0 otherwise
Y_{ink} = 1 if price level k is selected for the product i at echelon node n in time period t; 0 otherwise
Z_{ijnt} = 1 if product i immediately precedes product j produced at echelon node n in time period t; 0 otherwise
ZF_{ijnt} = 1 if there is product i in time period t-1 immediately precedes product j produced in time period t at echelon node n; 0 otherwise.

Continuous variables

$CET1_{nt}$ = time elapsed within time period t in a changeover starting in the previous time period at echelon node n
$CET2_{nt}$ = time elapsed within time period t in a changeover completing in the next time period at echelon node n
D_{int} = actual demand of product i sold at echelon node n in time period t
I_{int} = inventory of product i at echelon node n in time period t
I^D_{int} = inventory deviation to trajectory of product i at echelon node n in time period t
O_{int} = order index of product i produced in echelon node n in time period t
P_{int} = price of product i sold at echelon node n in time period t
PC_{int} = price change product i sold at echelon node n between time periods t-1 and t
PR_{int} = production amount of product i at echelon node n in time period t processing time of product i at echelon node n in time period t
$Q_{inn't}$ = flow of product i from echelon node n to n' in time period t
SV_{int} = sales volume of product i at echelon node n in time period t
SY_{intk} = auxiliary variable for linearization of $SV_{int} \cdot Y_{intk}$
$TotCC$ = total changeover cost
$TotPC$ = total production cost
$TotR$ = total revenue

$TotTC$ = total transportation cost
$TotUC$ = total unmet demand cost
U_{int} = unmet demand of product i sold at echelon node n in time period t
Φ_1 = total profit
Φ_2 = total weighted inventory deviation
Φ_3 = total weighted price change
Π = objective function

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Manuscript received Mar. 24, 2011, and revision received Dec. 10, 2011.